UNIVERSITÀ DEGLI STUDI DI MILANO

UNIVERSITÀ DI PAVIA

Ph.D. in ECONOMICS – Universities of Milan and Pavia

Static and dynamic optimization

Academic year 2016-17 – Fall Term

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Course description
The course introduces students to static and dynamic optimization. The analysis focuses on the development of some mathematical tools that are fundamental for advanced models in economic theory.

Course objectives
The course will offer an organic overview of some mathematical tools used in economic theory. In particular, static and dynamic optimization tools are introduced.

Course prerequisites
The usual contents of a basic calculus course and some basic notions in linear algebra are considered as prerequisites for the course.

Course organization: There are 8 lectures (20 hours) to be held in Via Pace, Aula B.

Course Assessment: The assessment is based on a written exam.

COURSE OUTLINE

1. **Linear algebra and quadratic forms.** Eigenvalues and eigenvectors, quadratic forms and their sign, constrained quadratic forms.

2. **Differential calculus.** Basic calculus on functions of several variables: partial derivatives and gradient, differentials, Taylor's formula, chain rules of differentiation. Concave functions and their properties. Quasiconcave functions. Implicit function theorems and their applications to comparative static analysis.

3. **Unconstrained and constrained optimization.** Unconstrained optimization problem: optimality conditions. Concave problems. Parametric unconstrained optimization problems. Optimization problem with equality constraints: Lagrangian function and optimality conditions. Concave problems. Parametric equality constrained problems. Optimization problems with inequality constraints: Kuhn-Tucker optimality conditions. Concave and quasiconcave problems. Parametric inequality constrained problems.

4. **Dynamical systems.** Differential and difference equations. Systems of differential and difference equations. Equilibrium solutions for dynamical systems and their stability. The linear case: solutions and stability of equilibrium solutions. Nonlinear case: linearization and Liapunov method.

5. **Dynamic optimization.** Optimal control and Maximum Principle. Transversality conditions. The case with infinite horizon. Autonomous problems.

6. **Dynamic programming.** Dynamic optimization and Bellman's principle.

Static optimization
Simon-Blume, *Mathematics for Economists*, WWNorton&Co.
Chapters: 14,15,16,17,18,19,21,23,30(30.3,30.4,30.5).
Other references:
Takayama, *Mathematical Economics*, CUP
Chiang, *Methods of Mathematical Economics*, Mc Graw Hill
De la Fuente, *Mathematical Methods and Models for Economists*, CUP
Beavis-Dobbs, *Optimization and Stability Theory for Economic Analysis*, CUP

Dynamic Optimization
Simon C.P. and Blume L.E., *Mathematics for Economists*, W.W. Norton and Company, Chapters 24 and 25;
A.C. Chiang, *Methods of Mathematical Economics*, Mc Graw Hill, Singapore, Chapters 13,14 and 18
Chiang, *Elements of Dynamic optimization*, Mc Graw Hill, Singapore, 2002, PART 3 Chapters 7, 8 and 9;

Leonard D. and Van Long N., *Optimal Control Theory and Static Optimization in Economics*, CUP Cambrige, 1992, Chapters 4,6,7 and 9;
Intrilligator M.D., *Mathematical Optimization and Economic Theory*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1971, Chapters 11, 13 and 14;
De La Fuente A., Mathematical Methods and Models for Economists, Cambridge University Press, Chapters 12;
M.I. Kamien, N,L. Schwartz, *Dynamic Optimization, The calculus of variations and optimal Control in Economics and Management*, North-Holland, PART II Sections 1-9 and 20;
N.L. Stokey, R.E. Lucas Jr., *Recursive Methods in Economic Dynamics*, Harvard University Press, Chapters 2, 4 and 5.1.

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