Monopolistic Competition when Income Matters

Paolo Bertoletti and Federico Etro

University of Pavia and Ca’ Foscari University of Venice

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Abstract

We study monopolistic competition with consumer preferences over differentiated goods characterized by a separable indirect utility rather than a separable direct utility as in the Dixit-Stiglitz model, with the CES case as the only common case. Examples include linear and log-linear direct demands. In equilibrium, an increase of the number of consumers is neutral on prices, but increases proportionally the number of firms, just creating pure gains from variety. Contrary to the Dixit-Stiglitz model with free entry, an increase in consumer income increases prices and more than proportionally the number of varieties if and only if the price elasticity of demand is increasing. We also discuss extensions of the basic setting to an outside good representing the rest of the economy, heterogeneous consumers, heterogeneous firms à la Melitz and endogenous quality. Finally, we provide an application to international trade generating pricing to market in a generalized Krugman model.

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The theory of monopolistic competition introduced by Chamberlin (1933) analyzes markets in which a large number of monopolistic firms choose prices independently and entry is free. The formalization proposed by Dixit and Stiglitz (D-S, 1977, Section I) and based on Constant Elasticity of Substitution (CES) preferences has become a workhorse model in modern economics. As well known, it implies constant markups and an endogenous number of firms that is proportional to both the number of consumers and their income (individual expenditure). Moreover, under firm heterogeneity, CES preferences imply that the market size has no selection effects (on the efficiency of active firms). These features have key consequences, for instance for the gains from international trade (Krugman, 1980 and Melitz, 2003) and for macroeconomic applications (see e.g. Blanchard and Kiyotaki, 1987 and Bilbiie et al., 2012).

From an empirical point of view, however, the CES model has some drawbacks. Primarily, it cannot account for the variability of markups across countries and trade conditions, and over the business cycle. There is indeed a consistent evidence that markups are higher in richer countries (see Alexandria and Kaboski, 2011 and Fieler, 2012), and there is also some evidence that they are variable over the business cycle (for instance, procyclical markups have been emphasized by Nekarda and Ramey, 2013). A recent interesting work by Simonovska (2013) has investigated international pricing of identical goods (online sales of goods shipped abroad) controlling separately for country population and income effects: she does not finds a significant impact of population on prices and estimates an elasticity of prices to per capita income between 0.05 and 0.11.

To account for these facts under monopolistic competition one has to depart from homothetic preferences. The general version of the additively separable direct utility function of D-S (1977, Section II) can be used as a source of variable markups (Krugman, 1979). However, it generates prices that can either decrease or increase in the number of consumers (Zhelobodko et al., 2012), implying an ambiguous impact of trade integration on welfare and ambiguous selection effects under firm heterogeneity (see Dhingra and Morrow, 2012, and Bertoletti and Epifani, 2012). Moreover, in spite of non-homotheticity, the D-S model with free entry generates a (rarely recognized in the literature) neutrality of the market structure with respect to income: markups and firm selection are unaffected by changes in consumers’ expenditure. In this paper, we propose an alternative model of monopolistic competition based on a different class of preferences, and argue that it can easily account for the stylized facts outlined above.

We assume that consumers’ preferences can be represented by an additively separable indirect utility function. Such “indirect additivity” amounts to assume that the relative demand of two goods does not depend on the price of other goods, while it depends on income (unless preferences are homothetic). It is thus different from the “direct additivity” exploited by D-S, for which the marginal rate of substitution between any two goods does not depend on the

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2 The empirical IO literature has shown that in concentrated markets a larger size may attract entry and reduce markups expanding firms’ output (see e.g. Campbell and Hopenhayn, 2005), but these strategic effects ought to disappear in markets characterized by many firms and monopolistic competition.
consumption of other goods. In fact, duality results (Hicks, 1969; Samuelson, 1969; Blackorby et al., 1978) tell us that the case of CES preferences is the only common ground (which requires homotheticity) of direct and indirect additivity. Under the latter, the number of goods provided in the market does not affect their substitutability and thus the price elasticity of demand, while income can affect this elasticity with crucial consequences.

Monopolistic competition with indirect additivity generalizes the neutrality of the number of consumers on the production structure which characterizes CES models, extending the pure gains from varieties (à la Krugman, 1980) to our entire class of preferences, as we show also in a two-country version of our setting. Moreover, we obtain markups that are variable in income/spending, with two appealing consequences. First, pricing to market emerges as a natural phenomenon: as long as demand is more rigid for richer consumers, we have higher markups in markets with higher individual income. Second, markups vary cyclically either when demand shocks affect income/spending or supply shocks affect marginal costs (i.e. firms’ productivity). Similar results hold also in a two-sector extension with an outside good representing the rest of the economy, when consumers are heterogenous in preferences and income, and even when firms are heterogeneous.

The comparative statics for business creation is also of interest. Consider the case where demand gets more rigid with income. Richer consumers induce firms to increase their markups, which attracts more than proportional entry of firms in the market. When firms are heterogeneous à la Melitz (2003), this establishes a Darwinian mechanism that is absent in the CES model: less productive firms enter in booms (when income increases) and exit during downturns (a sort of “cleansing effect” of recessions). Finally, suppose that firms can invest in the quality of their products: then, taking advantage of larger market shares, more productive firms tend to react to an increase in consumers’ income by endogenously offering products of higher quality sold at higher prices.

The work is organized as follows. In Section 1 we presents our baseline model of monopolistic competition. In addition to characterizing the endogenous entry equilibrium, we discuss indirect versus direct additivity, present some analytically workable examples of the former (leading to linear and loglinear demands) and recover the underlying direct utility functions. In Section 2 we extend the model in various directions and provide a welfare analysis. In Section 3 we apply our framework to an international trade model à la Krugman (1980) considering both costless trade between different countries and costly trade between identical countries. We conclude in Section 4. All the proofs are in the Appendix.

3 On the importance of non-homotheticity in trade models see Markusen (2013).

4 This is consistent with the so-called Linder hypothesis: for recent empirical support see Kugler and Verhoogen (2012).
1 The Model

Consider a market populated by \( L \) identical agents consuming a mass of \( n \) goods under the following symmetric and separable indirect utility function:

\[
V = \int_0^E v \left( \frac{P_j}{E} \right) dj,
\]

where \( E > 0 \) is the income of each agent to be spent in a continuum of differentiated varieties, and \( p_j > 0 \) is the price of variety \( j \).\(^5\) The expression on the RHS of (1) exploits the property of homogeneity of degree zero of the indirect utility, and crucially assumes additive separability, i.e. “indirect additivity”. To satisfy sufficient conditions for (1) being an indirect utility function while allowing for a possibly finite choke-off price \( s \), we assume that the indirect sub-utility \( v(s) \) is at least thrice differentiable, with \( v(s) > 0 \), \( v'(s) < 0 \) and \( v''(s) > 0 \) for any \( s < \bar{s} \), and that \( \lim_{s \to \bar{s}} v(s), v'(s) = 0 \), with \( v(s) = 0 \) for \( s \geq \bar{s} \). These assumptions imply that demand and extra utility are zero for a good that is not consumed.

The Roy identity provides the following direct demand function of each consumer for good \( i \):

\[
x_i(p_i, E, \mu) = \frac{v' \left( \frac{p_i}{E} \right)}{\mu},
\]

where

\[
\mu = \int_0^E v' \left( \frac{p_j}{E} \right) \frac{d_j}{E} dj.
\]

This generates the total market demand \( q_i = x_i(p_i, E, \mu)L \). Notice that \( \mu < 0 \) is the negative of the marginal utility of income, \textit{times} the income level \( E \).

Examples of (1) include simple cases such as the isoelastic function \( v(s) = s^{1-\theta} \) with \( \theta > 1 \), the exponential function \( v(s) = e^{-\tau s} \) with \( \tau > 0 \), or the “addilog” function \( v(s) = (a - s)^{1+\gamma} \) with \( a, \gamma > 0 \).\(^6\) Note that only if \( v(s) \) is isoelastic preferences are homothetic. Indeed, in such a case they are of the CES type, with indirect utility \( V = E \left( \int_j p_j^{1-\theta} dj \right)^{1/(1-\theta)} \) where \( \theta \) is the elasticity of substitution. By an important duality result (see Hicks, 1969; Samuelson, 1969; Blackorby et al., 1978), the case of CES preferences is the only one in which the class of preferences (1) satisfies “direct additivity” as well. That is, it is the only case in which preferences can also be represented by an additively separable direct utility function as the one assumed by D-S, \( U = \int u(x_j) dj \) with a well-behaved subutility \( u(\cdot) \). Therefore, the indirect utility (1) encompasses a different class of (non-homothetic) preferences whose corresponding direct utility functions are non-additive.

\(^5\)Using the wage as the \textit{numeraire}, \( E \) can also be interpreted as the labor endowment of each agent (in efficiency units).

\(^6\)Here the choke-off price \( \bar{s} = a \) can be made arbitrary large. Other examples include generalizations of the isoelastic function such as \( v(s) = (s + b)^{1-\theta} \), with \( \theta > 1 \), or “mixtures” such as \( v(s) = s^{1-\alpha} + s^{1-\theta} \) with \( \theta \neq \alpha > 1 \).
Suppose now that each variety is sold by a firm producing with constant marginal cost \( c > 0 \) and fixed cost \( F > 0 \). Accordingly, the profits of firm \( i \) can be written as:

\[
\pi(p_i, E, \mu) = (p_i - c) \frac{v'(\frac{p_i}{E}) L}{\mu} - F.
\] (4)

Notice that \( \mu \) is unaffected by the price choice of firm \( i \). The most relevant implication of this functional form is that the elasticity of the direct demand corresponds to the (absolute value of the) elasticity of \( v'(\cdot) \), which we define as

\[
\theta(s) \equiv -\frac{v''(s)}{v'(s)} s > 0
\]

This depends on the price as a fraction of income, \( p_i/E \), but is independent from \( \mu \) and \( L \). Instead, in the D-S case, the elasticity of inverse demand is uniquely determined by the consumption level.\(^7\) This difference is crucial for the Chamberlinian analysis of monopolistic competition with free entry because market adjustments (needed to restore the zero-profit condition) take place through shifts of demand due to changes in the mass of firms, which affect the marginal utility of income.

### 1.1 Equilibrium under monopolistic competition

Any firm \( i \) maximizes (4) with respect to \( p_i \). The FOC is:

\[
v'(\frac{p_i}{E}) + (p_i - c) \frac{v''(\frac{p_i}{E})}{E} = 0,
\] (5)

which requires that (locally) \( v''(s)s + v'(s) > 0 \), or equivalently \( \theta(s) > 1 \). Moreover we also assume satisfied the SOC, which requires \( 2\theta(s) > \zeta(s) \), where \( \zeta(s) \equiv -v'''(s)s/v''(s) \) is a measure of demand curvature. Notice that \( \theta'(s)s/\theta(s) = \theta(s) + 1 - \zeta(s) \), therefore \( \theta' > 0 \) if and only if \( \theta > \zeta - 1 \), in which case the demand becomes more elastic when the price goes up.\(^8\)

The FOC (5) can be rewritten as follows for the equilibrium price \( p^e \):

\[
\frac{p^e - c}{p''} = \frac{1}{\theta(\frac{p^e}{E})},
\] (6)

where the familiar expression for the Lerner index equates the reciprocal of our expression for demand elasticity.\(^9\) The price rule (6) shows that under indirect additivity the profit maximizing price is always independent from the mass of varieties supplied, because the latter does not affect the elasticity of demand (or the substitutability between goods). On the contrary, the optimal price grows with income if firms face a more rigid demand and vice versa, which provides a demand-side rationale for markups that are variable across markets (or over

\(^7\)In the general D-S model the (individual) inverse demand of variety \( i \) is given by \( p_i(x_i) = u'(x_i)/\lambda \), where \( \lambda \) is the marginal utility of income.

\(^8\)If demand is (locally) concave \( v''' > 0 \) the SOC is always satisfied and \( \theta' > 0 \). On the contrary, if demand is convex \( v''' < 0 \) we may have \( \theta' < 0 \).

\(^9\)To guarantee the existence of a solution to (6) we assume that \( \pi E > c \) (so that consumer willingness to pay is large enough) and that \( \lim_{x \to 0} \theta(s) = \pi E/(\pi E - c) \). Notice that the SOC guarantees uniqueness of the equilibrium.
the business cycle). Consider the realistic case of $\theta' > 0$: then, the model is consistent with typical forms of pricing-to-market, i.e., the same good should be sold at a higher price in richer (or booming) markets.\footnote{However, it is immediate to verify that $p'/E$ is always decreasing in income.} Similarly, under the same assumption a change in the marginal cost is transmitted (pass-through) to prices in a less than proportional way (undershifting). Summing up, we have:

**Proposition 1.** Under indirect additivity and monopolistic competition the equilibrium prices are always independent from the mass of active firms, and they increase in the income of consumers and less than proportionally in the marginal cost if and only if the demand elasticity is increasing in the price.

Since by symmetry the equilibrium profit is the same for all firms, and it is decreasing in their mass, we can characterize the endogenous market structure through the zero profit condition $(p - c)EL/np = F$. This and the pricing rule \( (6) \) deliver the free-entry mass of firms and the production size of each firm:

$$n^e = \frac{EL}{F\theta (\frac{a}{E})}, \quad q^e = F\frac{\theta (\frac{E}{c}) - 1}{c}. \quad (7)$$

The following proposition summarizes the comparative statics for $n^e$:

**Proposition 2.** Under indirect additivity, in a monopolistic competition equilibrium with endogenous entry the mass of firms increases proportionally with the number of consumers; it increases more than proportionally with the income of consumers and decreases with the marginal cost if and only if the demand elasticity is increasing in the price.

As a corollary, the equilibrium production of each firm $q_e$ is neutral with respect to the number of consumers, but it decreases with individual income and increases with the marginal cost if and only if the demand elasticity is increasing. To understand these comparative statics and their applications, it is convenient to think of changes in $L$ as changes in the scale of the economy, changes in $E$ as demand shocks on the disposable income of consumers and changes in $c$ as supply shocks to firms’ productivity. First of all, the impact of an increase in the number of consumers is always the same as under CES preferences: larger markets do not affect prices and production per firm, but they simply attract more firms without inducing any external effect on the price and production choices. This neutrality result and its key implications for the Krugman (1980) model extend from CES preferences to the entire class described by (1).\footnote{As an immediate consequence, increasing the population just induces gains from variety. This is a remarkable difference compared to the D-S model, where the existence of gains from variety can be guaranteed only when the equilibrium price is decreasing in the population (see Zhelobodko et al., 2012, and Dhingra and Morrow, 2012).}

An increase of the income/spending of consumers has complex implications. Consider the realistic case where higher income makes demand more rigid ($\theta' > 0$): in such a case, a positive demand shock induces firms to increase their markups and reduce sales accordingly, which in turn promotes business creation and increases the equilibrium number of active firms in a more than proportional way. Finally, consider an increase in firms’ productivity associated
with a reduction of the marginal cost (always assuming \( \theta' > 0 \)): lower costs are translated less than proportionally to prices, which increases the markups and attracts entry of new firms. Accordingly, and contrary to what happens with CES preferences, our general model suggests that demand and supply shocks generate additional processes of business creation/destruction. This should alter the dynamics of macroeconomic models with endogenous entry (see Etro and Colciago, 2010, and Bilbiie et al., 2012).

It is important to emphasize the differences of our setting with the general D-S model under non-homothetic preferences. Its free entry equilibrium can be summarized as follows:

\[
\frac{p^e - c}{p^e} = r \left( \frac{q^e}{L} \right), \quad n^e = \frac{ELr (q^e/L)}{F} \quad \text{and} \quad q^e = \frac{F \left[ 1 - r \left( \frac{q^e}{L} \right) \right]}{cr (q^e/L)},
\]

where \( r(x) = -u''(x)x/u'(x) \) is what Zhelobodko et al. (2012) call the “relative love for variety.” Here both the price and the quantity do depend on the population \( L \), which in turn affects non-linearly the number of firms: the exact impact depends on the sign of \( r'(x) \). A more surprising result (hardly noticed in the literature) is that the equilibrium price and firm size are independent from income \( E \) (not only with CES).\(^{12}\) In such a general D-S model, free entry eliminates any impact of income in spite of non-homotheticity, and markups cannot be affected by changes in consumer spending over the business cycle.

In conclusion, we remark that our microfoundation of demand can be applied to the case of a (finite) small number of firms to analyze Bertrand or Cournot competition.\(^{13}\) Then, a standard competition effect emerges: in particular, a larger or richer market attracts new firms, which intensifies competition and reduces the markups. As a consequence, the production of each firm increases and the equilibrium number of firms increases less than proportionally with the market size. This would match the evidence emphasized in the recent empirical literature for concentrated markets (see for instance Campbell and Hopenhayn, 2005, and Etro, 2013).

### 1.2 Examples and alternative microfoundations

Our results can be illustrated in simple examples with closed form solutions. For instance, consider the exponential function \( v(s) = e^{-\tau s} \), which generates the log-linear demand \( q_i = \text{const} \cdot e^{-\tau p_i/E} \). The free-entry equilibrium can be solved as follows:

\[
p^e = c + \frac{E}{\tau}, \quad n^e = \frac{E^2 L}{F(\tau + E)}, \quad q^e = \frac{E\tau}{E}.
\]

\(^{12}\)The reason of the different results is rooted in the market adjustment process. Since the profit expression with direct additivity is \( \pi = (u'(x)/\lambda - c)Lx - F \), where \( \lambda = \int u'(x) x dx / E \), there is a unique (symmetric) equilibrium (zero-profit) value of \( \lambda = (nu'(x)/x)/E \). On the contrary, under indirect additivity, there is a unique equilibrium value of \( L/\mu = LE/\{nv'(p/E)p \} \).

\(^{13}\)The strategic games generated under indirect additivity belong to the general class of aggregative games analyzed by Acemoglu and Jensen (2011) with fixed number of firms and Anderson et al. (2012) with endogenous entry.
Another example is based on the addilog case \( v(s) = (a - s)^{1+\gamma} \), that delivers the linear demand \( q_i = \text{const} \cdot (a - p_i/E) \) when \( \gamma = 1 \). In general this leads to:

\[
p^e = \frac{\gamma c + aE}{1 + \gamma}, \quad n^e = \frac{(aE - c) EL}{F(aE + \gamma c)}, \quad q^e = \frac{F(1 + \gamma)}{aE - c}.
\]

Notice that in both examples \( \theta' > 0 \): therefore growth in income makes demand more rigid, which leads firms to increase their prices and reduce their production, with a more than proportional increase in the number of firms. In addition, a marginal cost reduction is not fully translated on prices, which attracts more others through substitution between two varieties is affected by the consumption of all the directly separable as in the D-S model: (11) shows that the marginal rate of

\[\frac{\partial U}{\partial x_i} = \frac{\partial v}{\partial x_i} \frac{\partial q_i}{\partial x_i} \frac{\partial q_i}{\partial p} - \frac{\partial v}{\partial x_i} \frac{\partial q_i}{\partial x_i} \frac{\partial p}{\partial x_i} \]

In spite of this “additive” functional form, we know that preferences are not

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In spite of this “additive” functional form, we know that preferences are not

\[v(x_i) = x_i^\gamma \]
To compare alternative models of monopolistic competition, recall that indirect additivity amounts to assume that the consumption ratio of any two goods $i$ and $j$, $x_i/x_j$, does not depend on the price of any other good (notice that a similar assumption is implicit in all the empirical Logit models). This implies that in case of a common price $p_i = p_j$ the elasticity of substitution between varieties $i$ and $j$ does not depend either on the number of the other goods or on their prices, but it might depend on income. Under direct additivity of preferences, as well known, it is the marginal rate of substitution between any two goods $u'(x_i)/u'(x_j)$ which is independent from the consumption of other goods, leading to the property that their price ratio, $p_i/p_j$, is independent from the quantities of the other goods consumed. As an implication, the elasticity of substitution between varieties $i$ and $j$ in the case of a symmetric equilibrium depends only on the common consumption level.

More generally, one can notice that both the preferences represented by (1) and those in the D-S model belong to the larger family of symmetric preferences, that include fully non-separable utilities. With monopolistic competition the pricing rule is always determined by the demand elasticity, and under symmetric preferences the demand elasticity of the symmetric equilibrium must be a function of the (common) price-income ratio and of the number of varieties, say $\theta^e(p/E,n)$. Interestingly, such an equilibrium elasticity coincides with the common elasticity of substitution between varieties (see Bertoletti and Epifani, 2012). Accordingly, alternative assumptions on preferences generate different implications for $\theta^e$. As well known, CES preferences imply that $\theta^e$ is independent from both the price-income ratio and the number of goods provided in the market. Indirect additivity implies that $\theta^e = \theta^e(p/E)$ does not depend on the number of goods: in other words, the introduction of a new variety does not affect the substitutability between any of the existing goods. The D-S hypothesis of direct additivity implies that $\theta^e = \theta^e(np/E)$ depends on the product of the price-income ratio and the number of goods, as it is clear from (8) after noticing that in a symmetric equilibrium $q/L = E/(np)$: in other words, $\theta^e$ is equally sensitive to changes in $p/E$ or $n$. At the other extreme are models in which demand elasticity does not depend on the price-income ratio but just

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16 The elasticity of substitution between goods $i$ and $j$ is a logarithmic derivative of $x_i/x_j$ with respect to $p_i/p_j$: see Blackorby and Russell (1989) for a formal discussion of the concept.

17 If both these properties are assumed to hold, then symmetric preferences must be homothetic and of the CES type (see Blackorby et al., 1978).

18 This result may hold beyond indirect additivity. Consider the following non-separable symmetric indirect utility function:

$$V = \int_0^n (a - s_j)^2 dj - \frac{1}{n} \left[ \int_0^n s_j dj \right]^2.$$  

By Roy identity we obtain the demand function:

$$q_i = \text{const} \cdot [aE - (p_i - \bar{p})]$$

where $\bar{p} = \int p_j dj$ is the average price. This is essentially the functional form used in the textbook of Krugman et al. (2012, Ch. 8) to introduce monopolistic competition. However, here we have $\theta^e = p/aE$ and in equilibrium we obtain the price $p = c + aE$, which is increasing in income but independent from the number of consumers, with $n^e = aE^2 L/F(c + aE)$. 

9
depends on the number of varieties, i.e., \( \theta_e = \theta_e(n) \). Finally, notice that quasilinear preferences (linear with respect to an outside good) would generate an equilibrium elasticity which is independent from income, \( \theta_e = \theta_e(p, n) \): this is the case of the quadratic preferences adopted by Melitz and Ottaviano (2008), where \( \theta_e \) increases with respect to both its arguments, and of the Logit model (see Anderson et al., 2012), where \( \theta_e \) is actually independent from \( n \). Which one of these preferences is more plausible remains a key issue for future empirical research.

2 Extensions

2.1 Outside good and optimum product diversity

In this section we follow D-S (1977; Section II) and extend the model to an outside good representing the rest of the economy, as in many general equilibrium models with two sectors. Let us consider a sector producing a homogenous good under perfect competition and constant returns to scale, and a sector producing differentiated goods under monopolistic competition. We microfound the model with an indirect utility that has an intersectoral Cobb-Douglas form:

\[
V = \left( \frac{E}{p^0} \right)^\gamma \left( \int_0^n v \left( \frac{p_j}{E} \right) dj \right)^{1-\gamma},
\]

where \( p^0 \) is the price of the outside good and \( \gamma \in [0, 1] \); clearly (12) collapses to (1) for \( \gamma = 0 \). In the Appendix we show that the pricing rule for the differentiated goods remains the same as in (6), but the equilibrium mass of firms depends on the elasticity of the indirect sub-utility \( v \), defined as \( \eta(s) \equiv -v'(s)s/v(s) > 0 \), which reflects the relative importance of the differentiated goods.

**Proposition 3.** In a Cobb-Douglas two-sector economy with indirect additivity and monopolistic competition with endogenous entry in the differentiated sector, an increase in the number of consumers is neutral on prices and increases linearly the mass of firms, but higher income increases prices if and only if the demand elasticity is increasing in the price.

It is interesting to evaluate the welfare properties of this generalized equilibrium. As well known, firms do not fully internalize the welfare impact of their entry decision, which may lead to too many or too few firms. The constrained

\[19\] Under the Homothetic Translog preferences employed by Feenstra (2003) \( \theta_e(n) \) is monotonically increasing with \( n \). However, it is not difficult to find well-behaved preferences for which \( \theta_e(n) \) can be decreasing with respect to \( n \); an example is provided by the Homothetic Diewert expenditure function (see e.g. Varian, 1993, p. 209).

\[20\] A general specification of intersectoral preferences would not change the pricing rule, but the equilibrium number of firms would not necessarily remain linear in the number of consumers. Notice that the Cobb-Douglas model can be also reinterpreted as a two-period model where young agents have income \( E \) to be spent in the homogenous good or saved to consume the differentiated goods when old, with discount factor \( (1-\gamma)/\gamma \) and zero interest rate.

\[21\] See the original D-S paper, Kuhn and Vives (1999) and Dhingra and Morrow (2012) for
optimal allocation (controlling prices and number of varieties under a zero profit constraint) is derived in the Appendix and provides a simple comparison with the decentralized equilibrium for any $\gamma \in [0,1]$:

**Proposition 4.** In a Cobb-Douglas two-sector economy with indirect additivity, monopolistic competition with endogenous entry generates excess (insufficient) entry with too little (much) production by each firm if the elasticity of the indirect sub-utility $\eta(\cdot)$ is everywhere increasing (decreasing) in the price.

Paralleling D-S, an intuition for this result can be obtained by noticing that $\eta$ approximates the ratio between the revenue of each firm and the additional utility generated by its variety. If $\eta' > (\leq) 0$ they diverge and at the margin each firm finds it more profitable to price higher (lower), i.e., to produce less (more), than what would be socially desirable. This, in turn, attracts too many (too few) firms.\(^{22}\)

### 2.2 Heterogeneous consumers and income distribution

In this section we generalize our model to the case of consumers with different preferences and income. The model remains tractable and allows one to draw implications on the impact of income distribution on the market structure. We assume that there is a mass $L$ of consumers of different “types”. Types are distributed across the population according to the cumulative distribution function $C(h)$ with support $[0,1].^{23}$ The consumer of type $h$ has income $E_h$ and indirect utility function given by:

$$V_h = \int_0^\infty v_h \left( \frac{p_j}{E_h} \right) dj. \quad (13)$$

Each firm adopts a simple extension of the pricing rule (6) for homogenous consumers:

$$\frac{p^e - c}{p^e} = \frac{1}{\tilde{\theta} (p, C)} \text{ with } \tilde{\theta} (p, C) \equiv \int_0^1 \theta_h \left( \frac{p}{E_h} \right) \frac{E_h}{E} dC (h), \quad (14)$$

where $\tilde{\theta}$ is a weighted average of demand elasticities $\theta_h$, and the weight is the consumer of type $h$’s “fraction” of average income $\bar{E}$. Under free entry, the mass $L$ of consumer is again neutral, but the distribution of types is not.\(^{24}\)

**Proposition 5.** Under indirect additivity with heterogeneous consumers and monopolistic competition with endogenous entry, an increase in the mass of key references on this issue. To reach the first-best allocation would require marginal cost pricing and subsidies to the firms.

\(^{22}\)One may find it more reasonable the case in which the elasticity of the sub-utility decreases when income gets higher, which requires $\eta' > 0$. This is the case for the exponential and addilog cases: accordingly, they both imply excess entry.

\(^{23}\)We arrange consumer types in such a way that $h > k$ implies $E_h > E_k$, exclude any form of price discrimination (i.e., there is no market segmentation), and focus on the symmetric equilibrium.

\(^{24}\)A special case arises if preferences are of the exponential type, i.e., $v_h = e^{-\tau_h p/E_h}$. In such a case $\theta_h (p/E_h) = \tau_h p/E_h$ and therefore $\tilde{\theta} = \bar{\tau}/\bar{E}$, where $\bar{\tau} = \int_0^1 \tau_h dC (h)$: the market structure depends only on $\bar{\tau}$ and average income $\bar{E}$.
consumers is neutral on prices and increases linearly the number of firms. With identical preferences: 1) if the demand elasticity is increasing (decreasing) in the price, an improvement of the income distribution according to the likelihood-ratio dominance raises (decreases) prices and increases the mass of firms more (less) than proportionally to average income; 2) a mean preserving spread decreases prices and the mass of firms if and only if the demand elasticity is convex.

The impact of an improvement of income distribution is in line with the baseline model, but the impact of inequality is in general ambiguous. Consider identical preferences with a demand elasticity increasing and convex with respect to the price, as in the addilog example: in such a case a mean preserving spread of the income distribution increases the average demand elasticity that is expected by the firms, which reduces prices and induces business destruction.

2.3 Heterogenous firms and endogenous quality

Melitz (2003) has shown that under heterogeneous productivity of the firms and CES preferences there are no selection effects on the set of active firms when a market expands, for instance in a boom or when the country opens up to costless trade. However, under more general D-S preferences this neutrality holds for changes in income but not in the population, whose increase can give rise to ambiguous effects (depending on the shape of the relative love for variety). When prices are increasing with the size of consumption, an expansion of the market scale induces a selection effect, forcing the exit of the least productive firms, while less productive firms enter during a contraction of the market (see Zhlobodko et al., 2012 and Bertoletti and Epifani, 2012). In this section we show that under indirect additivity the number of consumers is always neutral, but it is income growth that has an impact, exerting an anti-selection effect as long as $\theta' > 0$: income growth attracts the entry of less productive firms, while low-productivity firms exit during downturns.

Following Melitz (2003), we assume that, upon paying a fixed entry cost $F_e$, each firm draws its marginal cost $c \in [c, \infty)$ from a continuous cumulative distribution $G(c)$ with $c > 0$. In the Appendix we show that the equilibrium price function $p(c)$ of an active $c$-firm is the same function of the marginal cost expressed in (6), that high-productivity firms produce more and are more profitable, and that they also charge lower markups if and only if $\theta' > 0$. Firms are active if their variable profit $\pi_v$ cover the fixed cost $F$, that is if they have a marginal cost below the cut-off $\hat{c}$ satisfying:

$$\pi_v(c) = \frac{[p(c) - \hat{c}] v'(p(c)/E)L}{\mu} = F.$$  \hspace{1cm} (15)

Moreover, the equilibrium must satisfy the endogenous entry condition:

$$\int_{\hat{c}}^{\infty} [\pi_v(c) - F] dG(c) = F_e :$$  \hspace{1cm} (16)

i.e., firms must expect zero profit from entering in the market. The two equations determine $\hat{c}$ and $\mu$ in function of $L$, $F$, $F_e$ and $E$, but in the Appendix
we show that a change in \( L \) produces no selection effects: an increase of the population is completely neutral on all the prices and on the productivity cutoff beyond which firms are active, even when preferences are not CES. Instead, changes in income induce novel effects on the structure of production. In the standard case where \( \theta' > 0 \), besides increasing the mark up of the infra-marginal firms, a rise of income creates an anti-selection effect attracting new and less efficient firms in the market:

**Proposition 6.** Under indirect additivity, monopolistic competition with endogenous entry and cost heterogeneity between firms, an increase in population is neutral on prices and on the productivity cutoff beyond which firms are active, even when preferences are not CES. Instead, changes in income induce novel effects on the structure of production. In the standard case where \( \theta' > 0 \), besides increasing the markup of the infra-marginal firms, a rise of income creates an anti-selection effect attracting new and less efficient firms in the market:

This result rationalizes a “cleansing effect” of recessions: these induce the exit of low-productivity firms leaving the high-productivity firms in the market, while expansionary shocks associated with higher spending attract low-productivity firms. Notice that such a cyclical process cannot be reproduced in the baseline Melitz model or in its D-S extension.

The heterogeneous costs model can be easily extended to take into account endogenous quality choices. This possibility has been recently explored to account for positive correlations of productivity with both quality and prices (see for instance Kugler and Verhoogen, 2012), but non-homothetic preferences are essential to explain a positive association of income with both quality and prices (the so-called Linder hypothesis). For simplicity, let us suppose that for a variety \( j \) with price \( p_j \) and quality \( k_j \geq 0 \) the sub-utility is now given by \( v_j = v(p_j/E)\varphi(k_j) \), where \( \varphi, \varphi' > 0 \) (higher quality increases both utility and demand without affecting demand elasticity), and \( \lim_{k \to 0} \varphi(k) = 0 \) (to avoid corner solutions). For simplicity, let us assume that a \( c \)-firm can produce goods of quality \( k \) at the marginal cost \( ck \), obtaining variable profits \( \pi_v = (p - ck)v(p/E)\varphi(k)L/\mu \).

The SOCs require \( 2\theta > \zeta, \xi \equiv k\varphi''/\varphi' < 2(\theta - 1), \) and \( \xi[\zeta - 2\theta] > (\theta - 1)[2\zeta - 3\theta] \). They imply that \( \theta' > 0 \) if \( \varphi' \geq 0 \).
more productive firms produce goods of higher quality. Moreover, they can even invest so much to sell them at higher prices compared to low productivity firms: this happens when the demand becomes more sensible to quality for products of higher quality (that is if $\varepsilon' > 0$). Finally, in line with the Linder hypothesis, higher income induces specialization in high quality, high price goods. Given the price and quality choices, variable profits are still increasing in productivity and income, and the free-entry mechanism operates as before.

3 Application to International trade

One of the main limits of the trade models based on monopolistic competition with CES preferences (see e.g., Krugman, 1980 and Melitz, 2003) is their inability in providing simple reasons why firms should adopt different markups in different countries. In fact, it is well known that pricing to market is a pervasive phenomenon: identical products tend to be sold at different markups in different countries and in particular prices appear to be positively correlated with per capita income (Alessandria and Kaboski, 2011) but not with country population (Simonovska, 2013). In this section we generalize the Krugman (1980) model to indirectly additive preferences and emphasize its implications for the structure of trade.\footnote{For empirical evidence in this direction see, for instance, Kugler and Verhoogen (2012).}

We consider trade between two countries sharing the same preferences (1) and technology, as embedded into the costs $c$ and $F$, which are given in labor units, but possibly with different numbers of consumers and different productivity (i.e., labor endowment in efficiency units). In particular, we assume that the labor endowments of consumers in the Home and Foreign countries are respectively $e$ and $e^*$, so that income levels are $E = we$ and $E^* = w^*e^*$. Accordingly, the marginal and fixed costs in the domestic and foreign countries are respectively $wc$ and $wF$ and $w^*c$ and $w^*F$. Let us assume that to export each firm bears an “iceberg” cost $d \geq 1$, and, as standard, let us rule out the possibility of parallel imports aimed at arbitraging away price differentials (i.e., international markets are segmented). Consider the profit of a firm $i$, based in the Home country, which can choose two different prices for domestic sales $p_i$ and exports $p_{zi}$:

$$
\pi_i = \frac{(p_i - wc)v'\left(\frac{L}{L'}\right)}{\mu} + \frac{(p_{zi} - wcd)v'\left(\frac{L^*}{L'^*}\right)}{\mu^*} - wF,
$$

(18)

where $\mu$ and $\mu^*$ are the Home and Foreign values of (3). A symmetric expression holds for a Foreign firm $j$, choosing prices $p^*_j$ and $p^*_{zj}$.

\footnote{Several recent papers have studied trade in multi-country models with non-homothetic preferences. Bertoletti and Epifani (2012) consider the general D-S model but focus on identical countries. Behrens and Murata (2012) and Simonovska (2013) use specific types of D-S preferences: the former paper assumes that market are not segmented while the latter deals with the case of international price discrimination. Finally, Fajgelbaum et al. (2011) consider products of different qualities within a Logit demand system.}
The optimal price rules for the Home firms are:

\[
\frac{p - wc}{p} = 1 \frac{\theta \left( \frac{v'}{E} \right)}{\theta \left( \frac{v'}{E^*} \right)}, \quad \frac{p_z - wc d}{p_z} = 1 \frac{\theta \left( \frac{v'}{E} \right)}{\theta \left( \frac{v'}{E^*} \right)},
\]

and the optimal price rules for the Foreign firms are:

\[
\frac{p^* - w^* c d}{p^*_z} = 1 \frac{\theta \left( \frac{v'}{E^*} \right)}{\theta \left( \frac{v'}{E^*} \right)}, \quad \frac{p^* - w^* c}{p^*} = 1 \frac{\theta \left( \frac{v'}{E^*} \right)}{\theta \left( \frac{v'}{E^*} \right)},
\]

so that four different prices emerge, with \( \mu = \frac{nv'}{n E} + \frac{n^* v'}{n^* E^*} \) and \( \mu^* = \frac{nv'}{n E^*} + \frac{n^* v'}{n^* E^*} \) in the symmetric equilibrium. The endogenous entry condition for the firms of the Home country reads as:

\[
\frac{(p - wc)v'}{\mu} \left( \frac{p}{E} \right)L + \frac{(p_z - wc d)v'}{\mu^*} \left( \frac{p_z}{E^*} \right)L^* = w F,
\]

and a corresponding one holds for the firms of the Foreign country:

\[
\frac{(p^* - w^* c)v'}{\mu^*} \left( \frac{p^*}{E^*} \right)L^* + \frac{(p^*_z - w^* c d)v'}{\mu} \left( \frac{p^*_z}{E} \right)L = w^* F.
\]

We can normalize the home wage to unity, \( w = 1 \), and close the model with the domestic resource constraint (or, equivalently, the labor market clearing condition):

\[
e L = n \left[ c x L + c d x z L^* + F \right],
\]

where \( x = v' (p/E)/\mu \) and \( x z = v' (p_z/E^*)/\mu^* \). This provides a system of seven equation in seven unknowns \((p, p_z, p^*, p^*_z, w^*, n \) and \( n^*)\). With non-homothetic preferences, population and productivity of a country have a distinct impact on the relative wages, with complex implications for price differentials and the structure of trade. However, we can study the main insights of the model focusing on the two cases traditionally analyzed in the literature: costless trade between countries different in population and income, and costly trade between identical countries.

The former case is obtained setting \( d = 1 \) with \( L \neq L^* \) and \( e \neq e^* \) and is characterized as follows:

**Proposition 7.** Under indirect additivity, monopolistic competition with endogenous entry and costless trade, firms adopt a higher price in the richer country independent from the number of consumers; opening up to trade shifts some firms from the richer to the poorer country if and only if the demand elasticity is increasing, and generates pure gains from variety.

Since costless trade induces factor price equalization (otherwise the zero-profit condition would not be satisfied in both countries), the price rules show immediately the emergence of pricing to market: under the standard assumption \( \theta' > 0 \), prices of identical goods are higher in the richer country because
demand is more rigid compared to the poorer country, and these prices are independent from the population of the two countries and their changes. Consumers enjoy new varieties produced abroad and bought at the same price of the domestic goods. Nevertheless, opening up to trade induces a redistribution of firms and production across countries which is absent in the Krugman (1980) model. Firms exporting to the poorer country sell there at a lower mark up and face entry of foreign firms in the domestic market: accordingly, they obtain lower variable profits, which leads to business destruction at home. The richer country is then characterized by a process of concentration in fewer and larger firms. On the contrary, business creation takes place in the poorer country, where firms start selling abroad at higher mark ups and reduce their size. Finally, total number of firms and prices of both countries remain the same as in autarky, therefore the gains from trade are always pure gains from variety as in Krugman (1980).

The second case we consider, the one of costly trade, is obtained by setting \( d > 1 \) with \( L = L^* \) and \( e = e^* \), and is characterized as follows:

**Proposition 8.** Under indirect additivity, monopolistic competition with endogenous entry and costly trade between identical countries, opening up to trade reduces the markup on the exported goods and the mass of firms in each country relative to autarky if and only if the demand elasticity is increasing.

Since countries face the same transport costs, wages and prices of the exported goods are equalized in both countries. However, the markup applied to goods sold at home and abroad is not the same when preferences are not homothetic. In particular, the markup (on the marginal cost \( cd \)) is lower for the exported goods if \( \theta' > 0 \), because firms undershift transport costs on prices. This shows a different form of pricing to market, which has the additional consequence of affecting the entry process compared to the neutrality of the Krugman (1980) model: as long as the average markup diminishes because of undershifting of the transport costs on export prices, opening up generates a process of business destruction in both countries. Gains from trade, therefore, do not derive from pure gains from variety (as in the Krugman model with transport costs), but potentially also from a downward pressure on the markup of the imported goods.

Notice that our setting breaks the neutrality of changes in trade costs and income on the structure of trade which holds in the Krugman (1980) model. First, a reduction in transport costs reduces the price of exports, but simultaneously increases their markups, which affects the number of firms as well. Second, under some additional conditions, richer countries trade relatively more between themselves than poorer countries, which is in line with the evidence (for instance see Fieler, 2011). More generally, we believe that our form of non-homothetic

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28 The export share on GDP can be derived as:

\[
\frac{\text{Exports}}{EL} = \frac{\nu'(p_z/E)p_z}{\nu'(p_z/E)p_z + \nu'(p/E)p}
\]

where prices satisfy (19). Under CES preferences this ratio is \( 1/(1+d^{\theta-1}) \), therefore the export share is independent from income. Under non-homotheticity, weak conditions satisfied with
preferences would allow future research to distinguish clearly between the impact of income and population on different aspects of trade (such as the emergence of multinationals, or the role of trade policy).

Finally, it is important to stress that, if preferences are not homothetic a form of pricing to market arises in a multi-country setting also under direct additivity, both with costless and costly trade (Markusen, 2013; Simonovska, 2013). The mechanism is simple: a larger income implies a larger individual consumption for each variety, which in turn affects markups. However, exactly for this reason, direct additivity also preserves the (ambiguous) impact of the number of consumers on markups. For example, the model of Simonovska (2013) predicts a negative relation between country population and prices for which she does not find support in the data, making her empirical findings more in line with the theoretical setting presented here.

4 Conclusions

We have studied monopolistic competition with non-homothetic preferences satisfying indirect additivity, an alternative (and not less plausible) assumption compared to the standard setting of Dixit and Stiglitz (1977). Under reasonable conditions (namely more rigid demand for higher income), it generates two main predictions that are in contrast with the D-S approach and await for additional empirical tests: the income of consumers should increase prices and more than proportionally the number of firms, while the number of consumers should be neutral on the market structure. Our framework is highly tractable and encompasses a number of analytically solvable cases as those with linear or log-linear direct demands. Therefore, we believe that it could be applied to analyze more complex frameworks usually considered exclusive territory for CES modeling: in other words, the adoption of indirect additivity opens the space to build closed and open economy models of imperfect competition with heterogenous firms and consumers, possibly in a dynamic general equilibrium framework.

References

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linear and log-linear demand (\(\zeta^* > 0\) is sufficient) guarantee that the export share increases with income because the relative demand for imported goods becomes more rigid (in line with the Linder hypothesis).
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Appendix

**Proof of Proposition 1.** By using \( \theta' p / (\theta E) = \theta + 1 - \zeta \geq 0 \) if and only if \( \theta + 1 \geq \zeta \), the result follows from the total differentiation of (6):

\[
\frac{\partial \ln p^e}{\partial \ln n} = 0, \quad \frac{\partial \ln p^e}{\partial \ln E} = \frac{\theta + 1 - \zeta}{2\theta - \zeta} \quad \text{and} \quad \frac{\partial \ln p^e}{\partial \ln c} = 1 - \frac{\theta + 1 - \zeta}{2\theta - \zeta},
\]

(24)

after noticing that \( 2\theta - \zeta > 0 \) from the SOC.

**Proof of Proposition 2.** Using the comparative statics in (24) and differentiating (7) we obtain:

\[
\frac{\partial \ln n^e}{\partial \ln L} = 1, \quad \frac{\partial \ln n^e}{\partial \ln E} = 1 + \frac{(\theta + 1 - \zeta)(\theta - 1)}{2\theta - \zeta} \quad \text{and} \quad \frac{\partial \ln n^e}{\partial \ln c} = -\frac{(\theta + 1 - \zeta)(\theta - 1)}{2\theta - \zeta}
\]

**Proof of Proposition 3.** By the Roy identity, the demand of each differentiated good is given by:

\[
x_i = \frac{\nu' (p_i/E)}{\int_0^\nu \left[ \nu' (p_j/E) p_j E - \gamma \nu' (p_j/E) \right] dj},
\]

and the profits of each firm \( i \) are given by:

\[
\pi_i = \frac{\nu' (p_i/E) (p_i - c) L}{\int_0^\nu \left[ \nu' (p_j/E) p_j E - \gamma \nu' (p_j/E) \right] dj} - F,
\]

(25)

where the denominator is unaffected by \( p_i \). It is immediate to verify that, independently from the value of \( \gamma \), each firm adopts the same pricing rule as in (6) and the comparative static properties of the profit-maximizing price \( p^e \) are then the same as in Proposition 1. The number of goods produced in the free-entry equilibrium can be derived as:

\[
n^e = \frac{EL (1 - \gamma) \eta (p^e/E)}{F \theta (p^e/E) [(1 - \gamma) \eta (p^e/E) + \gamma]},
\]

(26)
which now depends on \( \gamma \) and both the elasticities \( \theta \) and \( \eta \); changes in income affect prices and the allocation of expenditure between the differentiated goods and the outside one. Nevertheless, the number of firms is always proportional to \( L \). □

**Proof of Proposition 4.** We compare the market performance with a constrained optimal allocation which maximizes utility under a zero-profit condition for the firms. The problem boils down to:

\[
\max_{n,p} \left\{ n v \left( \frac{p}{E} \right) \right\} \quad \text{s.t.} \quad p = c + \frac{F}{x(p,n,E)} L,
\]

where

\[
x(p,n,E) = \frac{v' \left( \frac{p}{E} \right)}{n} \left[ v' \left( \frac{p}{E} \right) \frac{F}{E} - \frac{\gamma}{1 - \gamma} v \left( \frac{p}{E} \right) \right]
\]

is the symmetric demand of a variety. Notice that the zero-profit constraint implicitly defines \( n \) as a function of \( p \) that is continuous on \([c, \bar{p}]\), with \( n(c) = 0 \) and \( \lim_{p \to \bar{p}} n(p) \) which is a finite number. Accordingly, there must exist an internal optimum which satisfies the following FOCs:

\[
v \left( \frac{p}{E} \right) = -\rho \frac{F}{Lx^2} \frac{\partial x}{\partial n},
\]

\[
n v' \left( \frac{p}{E} \right) / E = -\rho \left[ 1 + \frac{F}{Lx^2} \frac{\partial x}{\partial p} \right].
\]

They imply

\[
\eta(p/E) = -\frac{2xL}{\gamma} + \frac{\partial \ln x(p,n,E)}{\partial \ln n} = -\frac{p - c}{p} + \frac{\partial \ln x(p,n,E)}{\partial \ln p},
\]

where \( \rho \) is the relevant Lagrange multiplier. It is easily computed that:

\[
\eta' = \eta \left( 1 - \theta + \eta \right) / \left( \frac{p}{E} \right).
\]

Since

\[
\frac{\partial \ln x(p,n,E)}{\partial \ln p} = \frac{\gamma \frac{\partial \ln x(p,n,E)}{\partial \ln p}}{\gamma + (1 - \gamma) \eta} - 1 = \frac{2\gamma - 1}{\gamma + (1 - \gamma) \eta},
\]

we obtain:

\[
\frac{p^* - c}{p^*} = \frac{\gamma + (1 - \gamma) \eta}{(1 - \gamma) \eta (1 + \eta) + \gamma \theta}
\]

(28)

Finally, the optimal mass of firms is:

\[
n^* = \frac{(1 - \gamma) \eta LE}{F \left[ (1 - \gamma) \eta (1 + \eta) + \gamma \theta \right]}
\]

(29)

Compare (28) with (6): the RHS of (28) is larger (smaller) than \( 1/\theta \) if (everywhere) \( \theta > 1 + \eta \). Since it follows from (27) that \( \eta' \leq 0 \) is equivalent to \( 1/ (1 + \eta) > 1/\theta \),
then (everywhere) \( \eta' \leq 0 \) is equivalent to \( p^c \leq p^* \), which in turn implies \( x^* \leq x^c \) by the zero-profit constraint. Using the fact that the RHS of (29) is larger (smaller) than the RHS of (26) if \( \eta' \) is smaller (larger) than zero we obtain \( n^c \leq n^* \), which completes the proof. \( \square \)

**Proof of Proposition 5.** The demand of a consumer \( h \) for good \( i \) can be written as \( x_{hi}(p_i, E_h, \mu_h) = v'_h(p_i/E_h)/\mu_h \), where \( \mu_h = \int_j v'_h(p_j/E_h)(p_j/E_h) \, dj \). The profits of firm \( i \) are given by:

\[
\pi(p_i) = (p_i - c)L \int_0^1 x_{hi}(p_i, E_h, \mu_h) \, dC(h) - F,
\]

which implies that the profit-maximizing price \( p_i \) satisfies the FOC:

\[
(p_i - c) \int_0^1 v''_h \left( \frac{p_i}{E_h} \right) \frac{1}{\mu_h} \, dC(h) + \int_0^1 v'_h \left( \frac{p_i}{E_h} \right) \frac{1}{\mu_h} \, dC(h) = 0.
\]

Symmetric pricing implies \( \mu_h = n v'_h(p/E_h)(p^c/E_h) \) and thus:

\[
\frac{p^c - c}{p^c} = -\int_0^1 \frac{E_h}{\mu_h} \, dC(h) = \frac{1}{\int_0^1 \theta_h \left( \frac{p}{E_h} \right) \frac{E_h}{\mu_h} \, dC(h)} = \frac{1}{\bar{\theta}(p^c, C)},
\]

where \( \theta_h \equiv -v''_h(p/E_h) p/v'_h(p/E_h) E_h \) and \( \bar{E} = \int_0^1 E_h \, dC(h) \). The price rule is thus independent from \( L \). Endogenous entry implies the following mass of firms:

\[
n^c = \frac{\bar{E} L}{\bar{\theta}(p^c, C)},
\]

which proves the first part of the proposition since \( L \) affects linearly \( n^c \).

To prove the second part, suppose that all consumers share the same preferences. It is then convenient to rewrite \( \bar{\theta} \) directly as \( \bar{\theta}(p, I) = \int^{E_1} E \theta(p/E) \frac{dI(E)}{E} \), where \( I(\cdot) \) is the income distribution function implied by \( C(\cdot) \). Consider a change in \( I \) according to likelihood ratio dominance: i.e., a change from \( I^0 \) to \( I^1 \) such that \( I^1(E)/I^0(T) \geq I^0(E)/I^0(T) \) for all \( E > T \), \( E, T \in [E_0, E_1] \), where \( I(\cdot) = I^1(\cdot) \) is the relevant density function. This implies that \( I^1 \) also (first-order) stochastically dominates \( I^0 \) (see for instance Shaked and Shanthikumar, 1994): thus, by a well-known result, this raises the average income (i.e., \( \bar{E}^1 > \bar{E}^0 \)). We can write:

\[
\bar{\theta}(p^c, I) = \int_{E_0}^{E_1} \theta \left( \frac{p^c}{E} \right) \, d\Phi(E; I) \quad \text{with} \quad \Phi(E; I) = \int_{E_0}^{E} \frac{T}{E} \, dI(T),
\]

where the cumulative distribution function \( \Phi \) has density \( \phi = \Phi' \). Notice that:

\[
\frac{\phi(E; I^1)}{\phi(T; I^1)} = \frac{E_{i1}(E)}{T_{i1}(T)} \geq \frac{E_{i0}(E)}{T_{i0}(T)} = \frac{\phi(E; I^0)}{\phi(T; I^0)} \quad \text{for all} \quad E > T, E, T \in [E_0, E_1].
\]

Accordingly, \( \Phi(E; I^1) \) dominates in terms of the likelihood ratio \( \Phi(E; I^0) \) and it must then be the case that \( \Phi(E; I^1) \leq \Phi(E; I^0) \) for all \( E \in [E_0, E_1] \) (i.e., the
former distribution first-order stochastically dominates the latter). It follows that when \( \theta \) \( (p/E) \) is a decreasing (increasing) function of \( E \) an improvement of income distribution according to likelihood-ratio dominance implies \( \tilde{\theta} (p, I^1) \leq (\geq) \tilde{\theta} (p, I^0) \) for all \( p \), which in turn decreases (increases) the equilibrium value of \( \tilde{\theta} \), and thus raises (decreases) the equilibrium price level and the mass of active firms more (less) than proportionally to the rise of average income.

Suppose now that \( I^1 \) is a mean-preserving spread of \( I^0 \). Then \( \bar{E}^1 = \bar{E}^0 = \bar{E} \). The function \( \theta \) \( (p/E) \) \( E/\bar{E} \) in the definition of \( \tilde{\theta} (p, I) \) is a concave (convex) function with respect to \( E \) if and only if \( \theta'' < (>) 0 \). By a standard result it must then be the case that \( \tilde{\theta} (p, I^0) > (<) \tilde{\theta} (p, I^1) \) when \( \theta'' < (>) 0 \). It follows that a mean-preserving spread decreases (increases) the equilibrium value of \( \tilde{\theta} \), and then raises (decreases) prices and the mass of firms when \( \theta \) is a concave (convex) function of the price. \( \square \)

**Proof of Proposition 6.** Let us start analyzing the price choices for the active firms. The variable profits of a \( c \)-firm are given by \( \pi_c = (p - c)v'(p/E)L/\mu \), where \( \mu = n \int_{E} \frac{1}{p} \) \( (p(c)/E)(p(c)/E) \) \( \frac{\partial g(c)}{\partial (c)} \) is independent of its price choice. Therefore, the pricing rule (6) applies to all firms. We denote with \( p = p(c) \) the profit-maximizing price of a \( c \)-firm, with \( x(c) = v'(p(c)/E)/\mu \) the individual consumption of its product, and with:

\[
\pi_c(c) = [p(c) - c]v'(p(c)/E)L/\mu
\]

its variable profit for given \( \mu \). Note that the optimal price of firm \( c \) does not depend upon \( L \) and \( \mu \) and follows the same comparative statics as in Proposition 1, with \( \partial \ln p(c)/\partial \ln c \gtrless 1 \) and \( \partial \ln p(c)/\partial \ln E \gtrless 0 \) if and only if \( \theta'' \gtrless 0 \). Moreover, the FOCs and SOCs for profit maximization imply the following elasticities with respect to the marginal cost (for given \( \mu \)):

\[
\frac{\partial \ln x(c)}{\partial \ln c} = -\theta (p(c)/E) \frac{\partial \ln p(c)}{\partial \ln c} < 0,
\]

\[
\frac{\partial \ln p(c)x(c)}{\partial \ln c} = - (\theta (p(c)/E) - 1)^2 / 2 \theta (p(c)/E) - \zeta (p(c)/E) < 0,
\]

\[
\frac{\partial \ln \pi_c(c)}{\partial \ln c} = 1 - \theta (p(c)/E) < 0.
\]

Accordingly, high-productivity (low-\( c \)) firms are larger, make more revenues, and are more profitable, as in Melitz (2003), but they charge lower (higher) markups if \( \theta \) is increasing (decreasing). In addition, again for given \( \mu \), we have the following elasticities with respect to income:

\[
\frac{\partial \ln x(c)}{\partial \ln E} = \theta (p(c)/E)^2 - \theta (p(c)/E) > 0,
\]

\[
\frac{\partial \ln p(c)x(c)}{\partial \ln E} = \theta (p(c)/E)^2 + 1 - \zeta (p(c)/E) / 2 \theta (p(c)/E) - \zeta (p(c)/E) > 1,
\]

\[
\frac{\partial \ln \pi_c(c)}{\partial \ln E} = \theta (p(c)/E) > 1.
\]
The size of each firm increases with $E$ (for given $\mu$), and revenues and profits increase more than proportionally. However, each price increases with respect to income only when $\theta' > 0$, and decreases otherwise. The set of active firms is the set of firms productive enough to obtain positive profits. Denote by $\hat{c}$ the marginal cost cutoff, namely the value of $c$ satisfying the zero cutoff profit condition $\pi_c(\hat{c}) = F$, or:
\[ [p(\hat{c}) - \hat{c}]' (p(\hat{c})/E) L = \mu F. \] (36)

The relation (36) implicitly defines $\hat{c} = \hat{c}(E, \mu F/L)$. Differentiating it yields:
\[
\frac{\partial \ln \hat{c}}{\partial \ln E} = \frac{\theta (p(\hat{c}))/E - 1}{\theta (p(\hat{c}))} > 0, \tag{37}
\]
\[
\frac{\partial \hat{c}}{\partial \mu} = \frac{\partial \ln \hat{c}}{\partial \ln F} = -\frac{\partial \ln \hat{c}}{\partial \ln L} = \frac{1}{1 - \theta (p(\hat{c}))/E} < 0. \tag{38}
\]

Endogenous entry of firms in the market implies that expected profits
\[
E[\pi] = \int_{\hat{c}}^{e} [\pi_c(c) - F] dG(c) \tag{39}
\]
must be equal to the sunk entry cost $F_\hat{c}$. The profits decrease when the absolute value of $\mu$ increases, that is $\partial E[\pi]/\partial \mu > 0$. Accordingly, the condition $E[\pi] = F_\hat{c}$ pins down uniquely the equilibrium value of $\mu$ as a function $\mu(E, L, F, F_\hat{c})$. In particular, using (36) the free entry condition can be written as:
\[
\int_{\hat{c}}^{e} \left\{ \frac{[p(c) - c]'}{p(c)} v'(p(c)/E) - 1 \right\} dG(c) = \frac{F_\hat{c}}{F}. \tag{40}
\]

The system \{(36), (40)\} can actually be seen as determining $\hat{c}$ and $\mu$ in function of $L$, $F$, $F_\hat{c}$ and $E$. The second equation fixes $\hat{c}$ and is independent from $L$, and the first one determines $\mu$ as linear with respect to $L$. The cut-off $\hat{c}$ is therefore independent of market scale, because $\mu$ proportionally adjusts in such a way to keep constant the ratio $L/\mu$ and thus the variable profit of the cut-off firm. As a consequence, as in Melitz (2003), a change in $L$ produces no selection effect, even when preferences are not CES. Also notice that a raise of $F$ requires an increase of $\mu$ less than proportional (otherwise the value of the expected variable profit would increase more than proportionally), and this in turn decreases $\hat{c}$ (a selection effect), while an increase of $F_\hat{c}$ by increasing the equilibrium value of $\mu$ raises $\hat{c}$ (an anti-selection effect). The impact of income $E$ is more complex. Since by (35) and (37) an increase of $E$ raises $E[\pi]$, it must decrease the equilibrium value of $\mu$. In particular:
\[
\frac{\partial \mu}{\partial E} \frac{E}{\mu} = -\frac{\partial E[\pi]}{\partial \mu} \frac{E}{\mu} = \overline{\theta}(\hat{c}) > 0, \tag{41}
\]

where
\[
\overline{\theta}(\hat{c}) = \left[ \int_{\hat{c}}^{e} \frac{1}{\theta(p(c)/E)} \frac{p(c)x(c)}{\int_{\hat{c}}^{e} p(c)x(c)dG(c)} dG(c) \right]^{-1}. \tag{42}
\]
is the harmonic mean of the $\theta$ values according to $G(\cdot)$ and $\hat{c}$. Computing the total derivative of $\hat{c}$ with respect to $E$ we obtain:

$$
\frac{d \ln \hat{c}}{d \ln E} = \left[ \frac{\partial \hat{c}}{\partial E} + \frac{\partial \hat{c}}{\partial \mu} \frac{\partial \mu}{\partial E} \right] \frac{E}{\theta} = \frac{\theta (p(\hat{c})/E)}{\theta (p(\hat{c})/E) - 1} + \frac{\bar{G}(\hat{c})}{\theta (p(\hat{c})/E) - 1},\tag{43}
$$

which is positive if and only if (everywhere) $\theta' > 0$: that is, in addition of increasing (decreasing) the mark up of the infra-marginal firms, a rise of $E$ creates an anti-selection (selection) effect if $\theta$ increases (decreases) with respect to the price. To close the model, the expected mass of active firms $n$ is determined by the budget constraint, requiring average expenditure to equal $E/n$, and thus:

$$
n = \frac{E}{\int_{\mathbb{C}} p(c)x(c)\frac{dG(c)}{\theta(c)}}.\tag{44}
$$

Since an increase of the mass of consumers $L$ affects proportionally $\mu$, and thus proportionally reduces individual consumption $x(c)$, it follows from (44) that it also proportionally increases the mass of varieties. □

**Proof of Proposition 7.** Let us assume $d = 1$. In such a case each firm faces the same demand functions, independently from the country in which it is based. However, the firms based in the Home country have a cost advantage (disadvantage) with respect to firms from the Foreign country if $w < (>) w^*$. Since a necessary condition for a monopolistic equilibrium with endogenous entry in both countries is $\pi = \pi^* = 0$, it follows that it must be $w/w^* = 1$. Accordingly, we can normalize the common wage to $w = w^* = 1$ (which restores the notation of the baseline model with $\bar{E} = e$ and $\bar{E}^* = e^*$), and conclude that in a symmetric equilibrium $p = p^*$ and $p^* = p_z$. This means that all firms adopt the same price in the same country, with:

$$
p - c = \frac{1}{\theta \left( \frac{1}{\bar{p}} \right)} \quad p^* - c = \frac{1}{\theta \left( \frac{1}{\bar{p}^*} \right)},\tag{45}
$$

where $p > p^*$ when $E > E^*$ if and only if $\theta' > 0$ (everywhere). The opening of costless trade has no impact on prices and mark ups relative to autarky, extending this property of the Krugman (1980) model to our entire class of indirectly additive preferences. From symmetry we infer that all firms have the same profit and using the price rules in (19) the zero-profit constraint provides the total mass of firms as:

$$
n + n^* = \frac{EL}{F\theta \left( \frac{1}{\bar{p}} \right)} + \frac{E^*L^*}{F\theta \left( \frac{1}{\bar{p}^*} \right)}.\tag{46}
$$

This is the sum of the masses of firms emerging under the autarky equilibrium in each separate country, say $n^a = EL/F\theta (p/E)$ and $n^a = E^*L^*/F\theta (p^*/E^*)$, therefore the total mass of firms remains the same. This implies that after opening up to trade welfare unambiguously increases because of the increase in the number of consumed
varieties. However, the mass of firms active in each country is not the same as in autarky. In fact, by using the resource constraints one can obtain:

\[
\frac{n}{n^*} = \frac{EL}{E^*L^*} \leq \frac{n^a}{n^a*} = \frac{EL\theta(p^*/E^*)}{E^*L^*\theta(p/E^*)} \text{ if } E > E^* \text{ and } \theta' \geq 0, \tag{47}
\]

where we used the fact that \( p/E \gtrless p^*/E^* \) if \( E > E^* \) and \( \theta' \gtrless 0 \). Since the total number of firms is constant, the number of firms in the rich (poor) country must decrease (increase). Finally, and clearly, the resource constraints of each country imply that whenever the domestic mass of firm increases (decreases) their size must reduce (raise).

**Proof of Proposition 8.** Let us assume \( d > 1 \) but \( L = L^* \) and \( e = e^* \). In such a case, all the equilibrium variables must be the same across countries by symmetry. Therefore we can again normalize \( w = w^* = 1 \), which implies that \( E = E^* \). The internal prices and the prices of exports must be the same in both countries, i.e., \( p = p^* \) and \( p_z = p_z^* \). These prices satisfy:

\[
\frac{p-c}{p} = \frac{1}{\theta\left(\frac{p}{E}\right)}, \quad \frac{p_z - dc}{p_z} = \frac{1}{\theta\left(\frac{p_z}{E}\right)}.
\]

By Proposition 1 we know that \( p_z > p \) and \( (p-c)/p \gtrless (p_z - dc)/p_z \) if and only if \( \theta' \gtrless 0 \). By symmetry the number of firms in each country is the same, say \( n \), but this does not need to be the same as in autarky, \( n^a = EL/F\theta(p/E) \). To find the number of firms in each country after opening up to trade, let us combine the free entry condition (21) with the price rules (19) to obtain:

\[
\frac{pxL}{\theta\left(\frac{px}{E}\right)} + \frac{p_z x z L}{\theta\left(\frac{p_z x z}{E}\right)} = F
\]

By using \( E = n (px + p_z x_z) \) the number of firms can be derived as follows:

\[
n = \frac{EL}{F} \left[ \theta\left(\frac{p}{E}\right)^{-1} \frac{px}{px + p_z x_z} + \theta\left(\frac{p_z}{E}\right)^{-1} \frac{p_z x_z}{px + p_z x_z} \right]. \tag{48}
\]

Notice that the parenthesis in (48) is a weighted average of \( 1/\theta(p/E) \) and \( 1/\theta(p_z/E) \). Under CES preferences this is a constant: the number of firms is the same as in autarky (remarkably, this is independent from the transport costs). Otherwise, since \( p_z > p \), we have \( n \gtrless n^a \) if and only if \( \theta' \gtrless 0 \). \( \square \)